

**A GENERALIZATION OF A CLASSICAL RELATION BETWEEN
 $J_{\nu+n}(z)$, $J_{\nu}(z)$ and $J_{\nu-1}(z)$ WITH COMMENTS ON
THE MODERN STATE AND TRENDS IN SPECIAL FUNCTIONS***

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Abstract

The classical formula connecting the Bessel function $J_{\nu+n}(z)$ with two contiguous functions $J_{\nu}(z)$ and $J_{\nu-1}(z)$ has been generalized to the case of three arbitrary associated functions. The derivation gives us an opportunity to compare traditional approaches to the theory of special functions with the operator factorization method based on the use of crucially new ideas and a novel powerful calculational technique.

Mathematics Subject Classification: 33C10

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1. Introduction

One view [1] holds that the state of affairs with the special functions of hypergeometric type does not leave too much to be desired ...Glorious tercentennial history of hypergeometric functions with great names of prolific contributors and strikingly beautiful results ...Renewed interest to traditional values of mathematics after many years of persuing “abstractions” ...Impetuous development of the theory of hypergeometric functions begun in the eighties of present century ... An outstanding unifying role of hypergeometric series in mathematics (and no less in physics)... All these statements made in [1] are truth, nothing but the truth yet not the whole truth! The dark side of the picture involves combersome and

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